OBSERVATION PLANNING IN IDENTIFYING DYNAMIC PROCESSES

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The observation plan concept [7] is considered as a generalization of the experiment-plan concept [8] for soluble problems in identifying processes with lumped parameters [1-3] and distributed ones [4-6].

I. Processes with Lumped Parameters. 1°. General model form. Let the model structure be known for a process by virtue of a hypothesis on the mechanism:

$$\begin{cases} \dot{x}_t = f(k, x, u, t), \; t \in S_t \subset [0, T], \; x \in X \subset R_+^n, \; k \in K \subset R_+^p, \\ x(0) = x_0 \in S_x \subset X, \; u \in S_u \subset U \subset R^s, \; f(\cdot) \in C^{\hat{Q}}([0, T] \times K \times X \times U), \; \hat{Q} \leqslant \infty, \\ y(t) = g(x) \in Y \subset C^{\hat{Q}}([T', T''] \times K \times X \times U; \; R^m), \; 0 \leqslant T' < T'' \leqslant T, \; m \leqslant n, \end{cases} \tag{1}$$

where the defining operator for the model M is: $K \times X \times U \longrightarrow Y$.

2°. Identification Problems. One can formulate the following problems for the model of (1):

a) estimation: $Mk = y_\delta$, $x_0^{*1} \in S_x$, $u^* \in S_u$, $k \in K$, $y_\delta = y(t) + \delta$, δ; δ is the observation error, $y_\delta(t) \in Y_\delta \subset C^Q([T', T''] \times K \times X \times U; \; R^m)$, $Q < \hat{Q}$;

b) observation problems [9]: $Mx_0 = y_\delta$, $u^* \in S_u$, $k^* \in K$, $x_0 \in X$, $y_\delta \in Y_\delta$;

c) joint observation and estimation [10]: $Mz = y_\delta$, $u^* \in S_u$, $z = \{x_0, k\} \in Z \equiv X \times K$, $y_\delta \in Y_\delta$.

The solution amounts to solving the extremal problem

$$\Phi_{L_p}(z) \equiv \rho_{L_p}(Mz, \; y_\delta) \underset{z}{\rightarrow} \min. \tag{2}$$

3°. Experiment Plan. For the model of (1), the experiment plan ε is the set of the plan net S and the plan measure (normalized weights for the net nodes S):

$$\varepsilon \equiv \{S, \; \alpha\}, \tag{3}$$

where $S = S_x \times S_u \times S_t \subset Д \equiv X \times U \times [0, T]$.

4°. Observation Plan. For problems a)-c), the data sources are not only the observations $y_\delta(t)$. One can take the graphs for the function and its derivatives (by the use of recovery methods) [11, 12] and various systems of functions [13, 14] $\{\varphi_i(t)\}^j \equiv \{j = 1\colon \delta(t_i - t), \; j = 2\colon t^{\alpha_i} \exp(-\beta_i t), \; j = 3\colon (\varphi_{i_1}(t), \; \varphi_{i_2}(t)) = \delta_{i_1 i_2}, \; \ldots, \; i = 1, \; 2, \; \ldots\} \subset \{\varphi(t)\}$ from a certain basic space $\{\varphi(t)\}$ for the following quantities derived from the basis of $y_\delta(t)$, $t \in [T', T'']$; $(\cdot, \; \varphi_i^j(t))$, $(\cdot, \; \varphi_i^j(\tau_x))$, $(\cdot, \; \varphi_i^j(\tau_u))$, , where (·) denotes the observation functions $\overset{(q)}{y}_{\delta_t}$, $\overset{(q)}{y}_{\delta_{\tau(x)}}$, $\overset{(q)}{y}_{\delta_{\tau(u)}}$, $\overset{(q)}{y}_{\delta_{(\cdot)}} = \dfrac{\partial^q y_\delta}{\partial(\cdot)^q}$, $q = 0, \; 1, \; 2, \; \ldots$, τ_x, τ_u^{2} being the arguments in the parametric specification of certain selected curves γ_x, γ_u in the sets X and U: $x_0(\tau_x) = \gamma_x(\tau_x)$, $u(\tau_u) = \gamma_u(\tau_u)$.

On this basis, the observation plan ε_V is the set

$$\varepsilon_V \equiv \{S_V, \; \alpha_V\}, \tag{4}$$

where $S_V = S_x \times S_u \times S_t \times S_\varphi$ is the observation net, $S_\varphi \subset \{\varphi(\cdot)\}$ is the function net, and α_V is the observation plan measure [7].

*Here and subsequently, an asterisk means that the quantity is fixed.
†Instead of double subscripts to τ, we use τ(·), e.g., τ(x) ≡ τ_x.

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The observation planning problem consists in choosing S_V and α_V on the basis of optimality criteria considered below. Then the solution to a)-c) on the optimal plan $\varepsilon_V{}^*$ amounts to solving an extremal problem analogous to (2), but considered in the generalized denumerable normed space $V_r = \Pi_l V^l_r,\ r=1, 2, \ldots,\ l=l(i, j, q)$,

$$\Phi_{V^l_r}(z) \equiv \rho_{V^l_r}(M_{|\varepsilon^*_V} z,\ y_\delta(\varepsilon^*_V)) \to \min_z, \tag{5}$$

where V^l_r is defined by the metric

$$\forall y_1,\ y_2 \in V^l_r,\ \rho_{V^l_r}(y_1,\ y_2) = \Big(\sum_{ijqv} \int_{T'}^{T''} \alpha^{tt}_{ijqv} |(\overset{(q)}{y}_{1,v_t} - \overset{(q)}{y}_{2,v_t},\ \varphi^j_i(t))|^r\, dt +$$

$$+ \int_{\tau'_x}^{\tau''_x} \alpha^{tx}_{ijqv} |(\overset{(q)}{y}_{1,v_t} - \overset{(q)}{y}_{2,v_t},\ \varphi^j_i(\tau_x))|^r\, d\tau_x + \ldots + \int_{\tau'_u}^{\tau''_u} \alpha^{uu}_{ijqv} |(\overset{(q)}{y}_{1,v_{\tau(u)}} - \overset{(q)}{y}_{2,v_{\tau(u)}},\ \varphi^j_i(\tau_u))|^r\, d\tau_u\Big)^{1/r}.$$

On running through the subscripts i, j, and q in some ordered fashion for the values in the metric $\rho_{V^l_r}(\cdot)$, we get a chain of embedded observation spaces

$$L_p \equiv V^1_r \supset V^2_r \supset \ldots \supset V^l_r \supset \ldots,\ p \equiv r = 1,\ 2,\ \ldots, \tag{6}$$

which implies ordering in the corresponding problems of a)-c) for the sets of quasisolutions [7, 15]

$$K_{V^1_r}(y_\delta(\varepsilon_V)) \supset K_{V^2_r}(y_\delta(\varepsilon_V)) \supset \ldots \supset K_{V^{l_f}_r}(y_\delta(\varepsilon_V)), \tag{7}$$

where $K_{V^l_r}(\cdot)$ is defined in accordance with [7, 13, 15] as

$$K_{V^l_r}(y_\delta(\varepsilon_V)) \equiv \{z \in Z : \rho_{V^l_r}(M_{|\varepsilon_V} z,\ y_\delta(\varepsilon_V)) \leqslant \Phi_{V^l_r}(\alpha_V,\ \delta)\},$$

$$l \equiv 1,\ 2,\ \ldots,\ l_f < \infty;$$

where l_j is the index to the last space in the chain of (6) for which the error δ still allows us to obtain the corresponding $V^{l_f}_r$ data sources.

II. Processes with Distributed Parameters. 1°. General process model. Let the model structure be known, the general form being

$$\begin{gathered} \dot{u}_t = F(k,\ u,\ \dot{u}_x,\ \ddot{u}_{xx},\ t),\ t \in [0,\ T],\ x \in X \subset R^l_+, \\ 1 \leqslant l < \infty,\ k \in K \subset R^p_+, \\ u^T = (u_1,\ \ldots,\ u_n),\ u_i(t,\ x) \in C^{\hat{Q}}(\tilde{D}),\ \tilde{D} \equiv [0,\ T] \times X, \\ \hat{Q} \leqslant \infty,\ \partial X = \bigcup_i \Gamma_i \equiv \Gamma, \end{gathered} \tag{8}$$

where u is the solution to (8), x is the geometrical-coordinate vector, and k is the physical-constant one. The physically realizable initial conditions $u(0,\ x) = \omega \in \Omega(X)$, and boundary conditions $\psi(\tilde{k},\ u(t,\ x)) = 0,\ \tilde{k} \in \tilde{K} \subset R^{\tilde{p}}_+,\ x \in \Gamma,\ t \in [0,\ T],\ \psi \in \Psi([0,\ T] \times \Gamma)$, are used with the values of the vectors k and k in the formulations for the problems a)-g) considered below, where they may or may not be known. Let $F(\cdot),\ g^{\cdot}(\cdot),\ \tilde{X} \subseteq X,\ \tilde{\Gamma} \subseteq \Gamma,\ \Omega_p \subseteq \Omega,\ \Psi_p \subseteq \Psi$ (Ω_p and Ψ_p are subsets of the parametrically specified initial and boundary conditions, including cases of problems with mobile boundaries $\Omega_p = \{\omega_p = \omega(p_\omega) : p_\omega \in P_\omega \subset R^{s_\omega}\},\ \Psi_p = \{\psi_p = \psi(p_\psi) : p_\psi \in P_\psi \subset R^{s_\psi}\}$, $n \leqslant s. < \infty$) be such that the trace vector functions in the model of (8) $y^\Gamma(t,\ x) = g^\Gamma(u(t,\ x))$ and $y^x(t,\ x) = g^x(u(t,\ x))$ have the necessary smoothness order, i.e.,

$$g^\Gamma(u(t,\ x)) \in Y \subset C^{\hat{Q}}([T',\ T''] \times \tilde{\Gamma} \times \Omega_p(\tilde{\tilde{X}}) \times \Psi_p([0,\ T] \times \tilde{\tilde{\Gamma}})\ K \times \tilde{K}),\ \tilde{\Gamma},\ \tilde{\tilde{\Gamma}} \subseteq \Gamma,\ \tilde{\tilde{X}} \subseteq X,$$

$$g^x(u(t,\ x)) \in Y \subset C^{\hat{Q}}([T',\ T''] \times \tilde{X} \times \Omega_p(\tilde{\tilde{X}}) \times \Psi_p([0,\ T] \times \tilde{\tilde{\Gamma}}) \times K \times \tilde{K}),$$

$$(g^{\cdot}(\cdot))^T = (g^{\cdot}_1,\ \ldots,\ g^{\cdot}_{m.}),\ m. \leqslant n.$$

The functions $g^{\Gamma}(\cdot)$ and $g^{x}(\cdot)$ are possible forms of the trace for a process whose experimental observation $y_\delta(t, x) = y(t, x) + \delta \in Y_\delta \in C^q([T', T''] \times \tilde{\Gamma} \times \tilde{X} \times \Omega_p(\tilde{X}) \times \Psi_p([0, T] \times \tilde{\Gamma})) \times K \times \tilde{K})$ contains an error δ. The model of (8) defines the operator M: $K \times \Omega(X) \times \Psi([0, T] \times \Gamma) \to Y$.

2°. Identification problems. For the model of (8) we can formulate the following problems:

a) estimation: $M\tilde{\tilde{k}} = y_\delta,\ \tilde{\tilde{k}} = \{k, \tilde{k}, \{k, \tilde{k}\}\},\ \tilde{\tilde{k}} \in \tilde{\tilde{K}} = K \times \tilde{K},\ \omega^* \in \Omega(X),\ \psi^* \in \Psi([0, T] \times \Gamma),\ y_\delta \in Y_\delta$;

b) observation problem: $M\omega = y_\delta,\ \psi^* \in \Psi(\cdot),\ \tilde{\tilde{k}}^* \in \tilde{\tilde{K}},\ \omega \in \Omega(X),\ y_\delta \in Y_\delta$;

c) recovery [16, 17]: $M\psi = y_\delta,\ \omega^* \in \Omega(X),\ k^* \in K,\ \tilde{k} \in \tilde{K},\ \psi \in \Psi(\cdot)$,

d) joint estimation and recovery: $M\{k, \psi\} = y_\delta,\ \omega^* \in \Omega(X),\ \tilde{\tilde{k}} \in \tilde{\tilde{K}},\ \psi \in \Psi(\cdot),\ y_\delta \in Y_\delta$;

e) joint observation and estimation: $M\{\omega, \tilde{\tilde{k}}\} = y_\delta,\ \psi^* \in \Psi(\cdot),\ \omega \in \Omega(X),\ \tilde{\tilde{k}} \in \tilde{\tilde{K}},\ y_\delta \in Y_\delta$;

f) joint observation and recovery: $M\{\omega, \psi\} = y_\delta,\ k^* \in K,\ \omega \in \Omega(X),\ \psi \in \Psi(\cdot),\ y_\delta \in Y_\delta$;

g) joint observation, recovery, and estimation: $M\{\omega, \psi, \tilde{\tilde{k}}\} = y_\delta,\ \omega \in \Omega(X),\ \psi \in \Psi(\cdot),\ \tilde{\tilde{k}} \in \tilde{\tilde{K}},\ y_\delta \in Y_\delta$.

Problems b)-g) can be solved not only in the formulation of pointwise determination of ω and ψ on a certain net but also in the parametric-identification formulation with parametric specification $\omega_p = \omega(p_\omega),\ \psi_p = \psi(p_\psi)$; problems a)-g) may be solved via the extremal problems

$$\Phi_{L_p}(z) = \rho_{L_p}(Mz, y_\delta) \to \min_z, \tag{9}$$

where $z = \{\tilde{\tilde{k}}, \omega, \psi, \{\tilde{\tilde{k}}, \omega\}, \ldots, \{\tilde{\tilde{k}}, \omega, \psi\}\}$.

3°. Experiment Plan. For the model of (8), the experiment plan ε is the following set:

$$\varepsilon \equiv \{S, \alpha\}, \tag{10}$$

where $S = S_\Gamma \times S_x \times S_t \times S_\Omega \times S_\psi \subset Д \equiv \tilde{\Gamma} \times \tilde{X} \times [0, T] \times \Omega_p(\tilde{\tilde{X}}) \times \Psi_p([0, T] \times \tilde{\tilde{\Gamma}})$; $S_\Gamma \subset \tilde{\Gamma} \subseteq \Gamma$ is a net of points at boundary Γ; $S_x \subset \tilde{X} \subseteq X$ is a net of points in region X, at the nodes of which one estimates $y_\delta(t, x)$; S_t is a net of observation times; $S_\Omega \subset \Omega_p(\tilde{X})$, $S_\psi \subset \Psi_p([0, T] \times \tilde{\tilde{\Gamma}})$ being nets of initial and boundary conditions correspondingly $S_\Omega \sim S_{p_\omega} \subset P_\omega$, $S_\psi \sim S_{p_\psi} \subset P_\psi$) on parametric specification). The planning problem, as in the case of model (1), consists of choosing S and α (or α in region D) to maximize optimality criteria considered below.

4°. Observation Plan. For the model of (8) and the problems of a)-g), we can use as data sources not only the observations $y_\delta(t, x)$ but also the following quantities taken by analogy with the model of (1): $(\cdot, \varphi_i^j(t)), (\cdot, \varphi_i^j(\tau_x)), (\cdot, \varphi_i^j(\tau_\omega)), (\cdot, \varphi_i^j(\tau_\psi))$, where (·) denotes the following functions: $\overset{(q)}{y}_{\delta_t}(t, x),\ \overset{(q)}{y}_{\delta_{\tau(x)}}(t, x),\ \overset{(q)}{y}_{\delta_{\tau(\omega)}}(t, x)\ \overset{(q)}{y}_{\delta_{\tau(\psi)}}(t, x),\ t \in S_t,\ x \in S_\Gamma \cup S_x,\ \gamma_x = x(\tau_x) \subset \tilde{\Gamma} \cup \tilde{X},\ \gamma_\omega = p_\omega(\tau_\omega) \subset S_{p_\omega} \subset P_\omega,\quad \gamma_\psi = p_\psi(\tau_\psi) \subset S_{p_\psi} \subset P_\psi$. These data sources enable us to introduce the generalized denumerably normalized space $V_r = \prod_l V_r^l$, for the solution of (9), where V_r^l is defined by the metric $\forall y_1, y_2 \in V_r^l,\ l = l(i, j, q),\ r = 1, 2, \ldots,$

$$\rho_{V_r^l}(y_1, y_2) = \Big(\sum_{ijqv} \int_{T'}^{T''} \alpha_{ijqv}^{tt} |(\overset{(q)}{y}_{1,v_t} - \overset{(q)}{y}_{2,v_t}, \varphi_i^j(t))|^r dt + \int_{\tau_x'}^{\tau_x''} \alpha_{ijqv}^{tx} |(\overset{(q)}{y}_{1,v_t} - \overset{(q)}{y}_{2,v_t}, \varphi_i^j(\tau_x))|^r d\tau_x + \ldots$$

$$\ldots + \int_{\tau_\omega'}^{\tau_\omega''} \alpha_{ijqv}^{t\omega} |(\overset{(q)}{y}_{1,v_t} - \overset{(q)}{y}_{2,v_t}, \varphi_i^j(\tau_\omega))|^r d\tau_\omega \ldots + \int_{\tau_\psi'}^{\tau_\psi''} \alpha_{ijqv}^{\psi\psi} |(\overset{(q)}{y}_{1,v_{\tau(\psi)}} - \overset{(q)}{y}_{2,v_{\tau(\psi)}}, \varphi_i^j(\tau_\psi))|^r d\tau_\psi\Big)^{1/r}. \tag{11}$$

Then the solution to a)-g) amounts to the solution on the chosen optimal observation plan $\varepsilon_V^* \equiv \{S_V, \alpha_V\}$, $S_V = S \times S_\omega \times S_\psi \times S_\varphi$, $\alpha_V = \{\alpha_{ijqv}^{tt}, \ldots, \alpha_{ijqv}^{tx}, \ldots, \alpha_{ijqv}^{t\omega}, \ldots, \alpha_{ijqv}^{\psi\psi}\}$ (in the generalized space V_r^l) for the following extremal problems:

$$\Phi_{V_r^l}(z) = \rho_{V_r^l}(M_{|\varepsilon_V^*} z, y_\delta(\varepsilon_V^*)) \to \min_z, \tag{12}$$

where z is as in (19) and condition (7) applied for (12) in relation to problems a)-g).

III. Criteria for Choosing the Optimal Observation-Plan Measure. The sets of quasisolutions $K_{V_r^l}(\cdot)$ for the problems of (5) and (12) in the Hilbert spaces can be approximated locally in the region of point $V_2{}^l$, $l = 1, 2, \ldots$, by an expansion of the form

$$K_{V_r^l}(y_\delta(\varepsilon_V)) \simeq \{z \in Z:\ \Phi_{V_2^l}(\alpha_V, z^*) + A(\alpha_V, z^*)\Delta z + \Delta z^T H(\alpha_V, z^*)\Delta z \leqslant \Phi_{V_2^l}(\alpha_V, \delta)\}; \tag{13}$$

$$A(\alpha_V, z^*) = \left(\frac{\partial\Phi.(\alpha_V, z^*)}{\partial z}\right)_{(p_z\times 1)};$$

$$H(\alpha_V, z^*) = \left(\frac{\partial^2\Phi.(\alpha_V, z^*)}{\partial z^2}\right)_{(p_z\times p_z)} = 2(I(\alpha_V, z^*) - J(\alpha_V, z^*));$$

$$I(\alpha_V, z^*) = \int_{(\cdot)\in Д}\left(\frac{\partial}{\partial z}(\overset{(q)}{y}, \varphi)\right)^T\left(\frac{\partial}{\partial z}(\overset{(q)}{y}, \varphi)\right)d\alpha_V(\cdot) \equiv I_V$$

being an information matrix;

$$J(\alpha_V, z^*) = \int_{(\cdot)\in Д}(\overset{(q)}{y} - \overset{(q)}{y}_\delta, \varphi)^T\frac{\partial^2(\overset{(q)}{y}, \varphi)}{\partial z^2}d\alpha_V(\cdot),$$

$$\Delta z = z - z^*, \quad \int_{(\cdot)\in Д} d\alpha_V(\cdot) = 1.$$

At the point z* (the minimum in the functional $\Phi_{V_2^l}(\alpha_V, z)$), the following condition applies [18]: $H(\alpha_V, z^*) = 2I(\alpha_V, z^*) + 0(\delta_V)$. The Hessian $H(\alpha_V, z^*)$ is related to the Gaussian curvature of the surface at point z* formed by the functional $\Phi_{V_2^l}(\alpha_V, z)$ in the space Z x R: $K(\alpha_V, z^*) = \prod_i^{p_z} \varkappa_i = |H(\alpha_V, z^*)|/|C(\alpha_V, z^*)|$, where $C(\alpha_V, z^*)$ is the first quadratic form of that surface at z*. Therefore, one can give a new geometrical representation for the D planning optimality criterion [8, 18]: $\det I(\alpha_V, z^*) \simeq \frac{1}{2}K(\alpha_V, z^*)$, which enables us to formulate more clearly the conditions for optimal conditioning in these problems and the specifications for the observation plan in order to meet these conditions. As the quasisolution sets of (7) for problems Ia)-Ic) and IIa)-IIg) are dependent on ε_V, we call observation plan ε_V^* locally even-optimal if for $y_\delta(\overset{*}{\varepsilon_V})$ and for space V_r^{l*} the following conditions are obeyed simultaneously:

1) the directions of the principal curvatures $\varkappa_i, i=\overline{1, p_z}$, coincide with the coordinate axes in Z formed by the unknown components of vector z;

2) the values of the principal curvatures are equal: $\varkappa_{\min}/\varkappa_{\max} = 1$; and

3) the Gaussian curvature is maximal: $\overline{K}(\overset{*}{\alpha_V}, z^*) = \max\limits_{\alpha_V:\int_D d\alpha_V = 1} K(\alpha_V, z^*)$.

The following quantitative criteria are proposed for obedience to the even-optimality observation-plan conditions (uncorrelated and equally accurate estimators, and minimality in the volume of the quasisolution set) for problems Ia)-Ic) and IIa)-IIg):

1) a criterion for estimator independence (plan orthogonality) [18]

$$W(I_V) = \det I_V/\prod_i^{p_z}\sum_j^{p_z}(I_{ij}^V)^2, \quad I_V = (I_{ij}^V)_{(p_z\times p_z)}, \tag{14}$$

2) a criterion for estimator equal accuracy (E criterion)

$$E(I_V) = \lambda_{\min}(I_V)/\lambda_{\max}(I_V), \ \lambda.(I_V) \tag{15}$$

being an eigenvalue of I_V, and

3) a criterion for minimality in the volume of the quasisolution set (D criterion)

$$G(I_V) = p_z / \max_{(\cdot)\in S_V} d(\overset{(q)}{y}, \varphi)_{|(\cdot)} \sim D(I_V) \equiv \det I_V, \tag{16}$$

where

$$d(\overset{(q)}{y}, \varphi)_{|(\cdot)} = Sp\left(\alpha_V(\cdot)\left(\frac{\partial}{\partial z}(\overset{(q)}{y}, \varphi)_{|(\cdot)}\right)^T I_V^{-1}\left(\frac{\partial}{\partial z}(\overset{(q)}{y}, \varphi)_{|(\cdot)}\right)\right), \ (\cdot)\in S_V.$$

Criteria W, E, and G take values in the range [0, 1]. The value zero for any of them corresponds to degeneracy in the problem, while the value one means obdeience to the condition for equal optimality in the plan corresponding to this criterion. On the basis of (14)-(16), we take the criterion for equal optimality as follows [18]

$$R(I_V) = (\mu_W W(I_V) + \mu_E E(I_V) + \mu_G G(I_V))/(\mu_W + \mu_E + \mu_G), \tag{17}$$

where μ are weighting factors chosen to suit the specific case.

Criteria W, E, G, and R may not attain the value one on any ε_V in a particular case. On the other hand, for each value of W**, E**, G**, R** realized on a certain observation plan ε_V^{**}, there may exist a set of different observation plans $\{\varepsilon_V^{**}\}$, including those realized on different nets of the functions $\{S_{\varphi,1}, S_{\varphi,2}, \ldots\}$. Therefore, one can say that there are classes of locally equivalent observation spaces $V_r^l/_{W^{**},\ldots,R^{**}}$ for a given problem (in the sense of a selected criterion or criteria), and also that there is an optimal pair (also not unique) of spaces $\{Z, V_r^{l*}\}$, where by V_r^{l*} one understands any such observation space in which there exist a net S_V* and a measure α_V* on this net such that $\Theta(I_V(\alpha^*_{V_r^{l*}}, z^*)) = \max_{\alpha_{V_r^l}} \Theta(I_V(\alpha_{V_r^l}, z^*))$,

where $\Theta = \{W, E, G, R\}$.

NOTATION

t, time; x, phase locus coordinates (part I), geometrical coordinates (part II); X, set of phase loci (part I), geometrical domain (part II); k, $\tilde{k}$, physical constants vector; u, control vector (part I), vector function (solution to system (8)), (part II); S_t, time net; S_x, initial condition net (part I), domain X net (part II); S_u, control net; M, model operator; $K, \tilde{K}, \tilde{\tilde{K}}$, set of values for the physical constants; U, control set; Y, set of process trace loci; y_δ, observation on the trace; δ, error; z, set of constants and initial conditions (part I), set of constants, initial and boundary conditions (part II); Φ(·), discrepancy functional; ρ(·), space metrics; ε, experimental design; S, experimental design net; α, experimental design measure; D, region of experiments; φ(·), function $\{\varphi(t)\}$, basic space of functions φ(·); τ_x, τ_u, arguments of parametric curves in X and U; ε_V, observation design; S_V, observation design net; α_V, observation design measure; V_r^l, generalized space with metrics of (5) and (11); $K(y_\delta(\varepsilon))$, set of quasisolutions; ω, initial conditions; Ω(·), set of initial conditions; ψ, boundary condition; ψ(·), set of boundary conditions; $\omega(p_\omega)$, $\psi(p_\psi)$, parameter-dependent initial and boundary conditions; S_ω, initial condition net; S_ψ, initial boundary condition net; S_φ, net of basic-space functions; A(·), discrepancy functional gradient; H(·), Hesse matrix of discrepancy functional; I_V, information matrix; J_V, observation-dependent part of Hesse matrix H(·); $C(\alpha_V, z^*)$, first quadratic form of the surface; $K(\alpha_V, z^*)$, Gaussian curvature of discrepancy functional surface; $\varkappa_i$, principal curvatures of discrepancy functional surface; W, E, G, and R, design optimality criteria; μ, weighting factors.

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